The Impacts of Weighting Functions on the Robust Performance of a Multi-Axial Piezoelectric Stage

Fu-Cheng Wang* and Ru-Chang Wu

Department of Mechanical Engineering, National Taiwan University, Taiwan

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*Corresponding author: fcw@ntu.edu.tw

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Abstract: This paper applies robust control strategies to a piezoelectric transducer (PZT) stage, and investigates the influences of weighting functions on the system performance. The designed controllers are implemented for experimental verification, and are shown to significantly improve aspects of the system performance, such as settling time, overshoot, and root-mean-square error. The developed loop-shaping design procedures can be directly applied to general PZT systems.

Keywords: robust control; loop-shaping; weighting; gap; stability margin; PZT; nano-stage

Introduction

Advancements in technology and increased requirements for precision have prompted the application of the piezoelectric transducer (PZT) to achieve precision positioning because of its advantageous properties, such as high bandwidths and large driving forces. However, the nonlinear characteristics (e.g., hysteresis) might influence system performance and decrease the extent of precision that can be achieved. Therefore, many researchers have applied control techniques for improving the positioning precision of PZTs. For example, Kung and Fung used a neural network and proportional-integral (PI) control for a PZT actuator that achieved a maximum deviation of 0.8% [1]. [2] applied proportional-derivative (PD) and lead-lag control to drive a PZT and achieved a maximum error of $2.5\mu\text{m}$. [3] integrated a long-range stage driven by ball screw mechanism and a short-range nano-precision PZT stage to accomplish a long travel table with linear accuracy of 10nm. The PZTs' nonlinear effects (e.g., hysteresis) [4, 5] have been modeled by many nonlinear examples, such as the Bouc-Wen model [6], the Preisach Model [7], the Maxwell slop model [8] and the Prandtl-Ishlinskii Hysteresis Inverse model [9, 10], to describe PZT dynamics. However, these models might be too complicated for control design. Sethi et al. applied loop-shaping and LQR techniques to control a flexible

beam, and showed that loop-shaping design can achieve better performance [11].

Therefore, in our previous work [12, 13], we adopted the concepts of gap-metrics [14] to represent PZTs as linear models and we regarded the nonlinearities as system uncertainties. We then applied robust control methodologies to restrain system uncertainties and noise, and achieved a precision of 2nm for a two-axial PZT stage. During the design process, we noted that the selection of weighting functions for robust control could significantly influence the designed controllers and the achievable system performance. Therefore, in the present paper, we further discuss the impacts of weighting functions on the performance of PZT systems that employ robust control techniques.

The remainder of paper is organized as follows: Section 2 introduces the PZT stage and derives its transfer functions by identification techniques. Section 3 describes robust control methodologies and applies different weighting functions for controller design. Section 4 implements the designed robust controllers and verifies the system performance through experiments. We also design a standard proportional-integral-derivative (PID) controller for performance comparison. The results show that the designed robust controllers can achieve much better performance than an ordinary PID control. Lastly, we draw conclusion in Section 5.

System Description

The two-axial PZT stage shown in Figure 1 includes three layers: the loading stage on the top, the X-axis stage in the middle, and the Y-axis stage at the bottom. The specifications for the PZT stage are listed in Table 1. We implemented PZTs to control the movements of the stages, and we measured stage positions using Mercury 5800 encoders [15], whose resolution is 1.22nm.

We constructed the control structure shown in Figure 2 for the PZT stage. We used the PXI-8108 [16] and Labview™ for data acquisition and a voltage amplifier [17] for driving the PZTs because the output voltage of PXI-8108 is limited to $\pm 10V$. In addition, we could increase the position resolution of the encoders four-fold (i.e., by 0.31 nm) using software [18].

We tested the characteristics of the PZT stage by supplying the input voltages shown in Figure 3(a) and we measured the output displacements shown in Figure 3(b). The X- and Y-axis are basically decoupled in these figures, so we could design independent controllers for the two axes, as described in Section 3.

System Identification

We considered the nonlinear characteristics of the PZT stage by applying identification techniques to obtain the system transfer functions at different operating conditions. First, we generated swept sinusoidal input voltages of 0.1~100Hz, with magnitudes varying from 0.5V to 4.5V at an interval of 0.5V, to drive the PZTs. We then measured the output displacements and applied subspace system identification methods [19] to derive the stage models. These transfer functions are listed in Table 2. Figure 4 shows the maximum singular values of these transfer functions, where we noted the system variations caused by PZT hysteresis or other nonlinear effects. Therefore, we applied robust control techniques for precision positioning because it has profound theorems for improving performance for systems with uncertainties.

Fu-Cheng Wang received the B.S. and M.Sc. degrees in mechanical engineering from National Taiwan University, Taipei, Taiwan, in 1990 and 1992, respectively, and the Ph.D. degree in control engineering from Cambridge University, Cambridge, U.K., in 2002. From 2001 to 2003 he worked as a Research Associate in the Control Group of the Engineering Department, University of Cambridge, U.K. Since 2003 he has been with the Control Group of Mechanical Engineering Department at National Taiwan University, where he is now a Professor. His research interests include robust control, fuel cell system control, inerter research, suspension control, and system integration and control.

Ru-Chang Wu received the B.S. degree in bio-industrial mechatronics engineering from National Chung-Hsing University, Tai-Chung, Taiwan in 2011. He is now with the Control Group of Mechanical Engineering Department at National Taiwan University.

Figure 1. The two-axial PZT stage: (a) illustration and (b) Photo.

Figure 2. The control structure.

Table 1. Specifications of the PZT actuator.

Type	osi-stack (+)	U_{max}	150V	PSt	150/5x5/20
Ceramic cross-section				5x5 (mm ²)	
Max. voltage range				-30~150(V)	
travel				18 (μm)	
Resonance frequency				50(kHz)	
Max. load force				2000(N)

Figure 3. The testing signals: (a) The input voltages and (b) The output displacements.

Figure 4. Maximum singular values: (a) G_x^i and (b) G_y^i .

Table 2. identified transfer functions.

X-axis	Y-axis
$G_x^1 = \frac{2.856s^3 + 2409s^2 + 4.393 \times 10^6 s + 4.403 \times 10^7}{s^4 + 1477s^3 + 1.709 \times 10^6 s^2 + 1.974 \times 10^9 s + 1.679 \times 10^{10}}$	$G_x^1 = \frac{2.856s^3 + 2409s^2 + 4.393 \times 10^6 s + 4.403 \times 10^7}{s^4 + 1477s^3 + 1.709 \times 10^6 s^2 + 1.974 \times 10^9 s + 1.679 \times 10^{10}}$
$G_x^2 = \frac{48.63s^3 + 7.194 \times 10^4 s^2 + 8.517 \times 10^7 s + 3.024 \times 10^9}{s^4 + 3494s^3 + 9.771 \times 10^6 s^2 + 2.648 \times 10^{10} s + 7.443 \times 10^{11}}$	$G_x^2 = \frac{48.63s^3 + 7.194 \times 10^4 s^2 + 8.517 \times 10^7 s + 3.024 \times 10^9}{s^4 + 3494s^3 + 9.771 \times 10^6 s^2 + 2.648 \times 10^{10} s + 7.443 \times 10^{11}}$
$G_x^3 = \frac{10.58s^3 - 1.393e \times 10^4 s^2 + 1.753 \times 10^7 s + 4.865 \times 10^8}{s^4 + 2009s^3 + 6.42 \times 10^6 s^2 + 6.972 \times 10^9 s + 1.427 \times 10^{11}}$	$G_x^3 = \frac{10.58s^3 - 1.393e \times 10^4 s^2 + 1.753 \times 10^7 s + 4.865 \times 10^8}{s^4 + 2009s^3 + 6.42 \times 10^6 s^2 + 6.972 \times 10^9 s + 1.427 \times 10^{11}}$
$G_x^4 = \frac{48.63s^3 + 7.194 \times 10^4 s^2 + 8.517 \times 10^7 s + 3.024 \times 10^9}{s^4 + 3494s^3 + 9.771 \times 10^6 s^2 + 2.648 \times 10^{10} s + 7.443 \times 10^{11}}$	$G_x^4 = \frac{48.63s^3 + 7.194 \times 10^4 s^2 + 8.517 \times 10^7 s + 3.024 \times 10^9}{s^4 + 3494s^3 + 9.771 \times 10^6 s^2 + 2.648 \times 10^{10} s + 7.443 \times 10^{11}}$
$G_x^5 = \frac{-13.22s^3 - 5276s^2 + 3.144 \times 10^5 s + 4.487 \times 10^6}{s^4 + 1485s^3 + 4.565 \times 10^6 s^2 + 1.379 \times 10^8 s + 8.963 \times 10^8}$	$G_x^5 = \frac{-13.22s^3 - 5276s^2 + 3.144 \times 10^5 s + 4.487 \times 10^6}{s^4 + 1485s^3 + 4.565 \times 10^6 s^2 + 1.379 \times 10^8 s + 8.963 \times 10^8}$
$G_x^6 = \frac{-13.3s^3 + 631.5s^2 + 2.024 \times 10^5 s + 1.3 \times 10^6}{s^4 + 1922s^3 + 4.986 \times 10^6 s^2 + 6.528 \times 10^7 s + 2.141 \times 10^8}$	$G_x^6 = \frac{-13.3s^3 + 631.5s^2 + 2.024 \times 10^5 s + 1.3 \times 10^6}{s^4 + 1922s^3 + 4.986 \times 10^6 s^2 + 6.528 \times 10^7 s + 2.141 \times 10^8}$
$G_x^7 = \frac{-14.45s^3 + 5647s^2 + 4.123 \times 10^5 s + 2.941 \times 10^6}{s^4 + 2375s^3 + 5.789 \times 10^6 s^2 + 1.138 \times 10^8 s + 3.772 \times 10^8}$	$G_x^7 = \frac{-14.45s^3 + 5647s^2 + 4.123 \times 10^5 s + 2.941 \times 10^6}{s^4 + 2375s^3 + 5.789 \times 10^6 s^2 + 1.138 \times 10^8 s + 3.772 \times 10^8}$
$G_x^8 = \frac{-11.41s^3 - 2.591 \times 10^4 s^2 + 5.529 \times 10^7 s + 5.438 \times 10^8}{s^4 + 2314s^3 + 9.769 \times 10^6 s^2 + 1.765 \times 10^{10} s + 6.365 \times 10^{10}}$	$G_x^8 = \frac{-11.41s^3 - 2.591 \times 10^4 s^2 + 5.529 \times 10^7 s + 5.438 \times 10^8}{s^4 + 2314s^3 + 9.769 \times 10^6 s^2 + 1.765 \times 10^{10} s + 6.365 \times 10^{10}}$
$G_x^9 = \frac{-4.572s^3 - 1.267 \times 10^4 s^2 + 5.065 \times 10^7 s + 4.962 \times 10^8}{s^4 + 2204s^3 + 9.032 \times 10^6 s^2 + 1.585 \times 10^{10} s + 5.371 \times 10^{10}}$	$G_x^9 = \frac{-4.572s^3 - 1.267 \times 10^4 s^2 + 5.065 \times 10^7 s + 4.962 \times 10^8}{s^4 + 2204s^3 + 9.032 \times 10^6 s^2 + 1.585 \times 10^{10} s + 5.371 \times 10^{10}}$

Controller Design

We reduced the influences of PZT nonlinearities on stage performance by designing robust controllers for the PZT stage. The following Small Gain Theorem is usually applied for robust stability analysis:

Theorem: Small Gain Theorem [20]

Suppose $M \in RH_\infty$ and let $\gamma > 0$, the system of Figure 5 is well posed and internally stable for all $\Delta(s) \in RH_\infty$ with: (a) $\|\Delta\|_\infty \leq 1/\gamma$ if and only if $\|M\|_\infty < \gamma$; (b) $\|\Delta\|_\infty < 1/\gamma$ if and only if $\|M\|_\infty \leq \gamma$, where $\|M\|_\infty$ is the H_∞ norm of system M .

Figure 5. The Small Gain Theorem.

Suppose a nominal plant G_0 has a *normalized left coprime factorization* $G_0(s) = \tilde{M}(s)^{-1} \tilde{N}(s)$, where $\tilde{M}, \tilde{N} \in RH_\infty$ and $\tilde{M}\tilde{M}^* + \tilde{N}\tilde{N}^* = I$. Assume a perturbed system G_Δ can be described as:

$$G_\Delta = (\tilde{M} + \Delta_{\tilde{M}})^{-1} (\tilde{N} + \Delta_{\tilde{N}}), \quad (1)$$

with $\|[\Delta_{\tilde{N}} \ \Delta_{\tilde{M}}]\|_\infty < \varepsilon$, and $\Delta_{\tilde{M}}, \Delta_{\tilde{N}} \in RH_\infty$, as shown in Figure 6(a). When $r=0$, the closed-loop system can be rearranged as Figure 6(b). Therefore, from Small Gain Theorem, the system is internally stable for all uncertainties $\Delta = [\Delta_{\tilde{N}} \ \Delta_{\tilde{M}}]$ with $\|\Delta\|_\infty < \varepsilon$ if and only if

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G_0 K)^{-1} \tilde{M}^{-1} \right\|_\infty = \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G_0 K)^{-1} \begin{bmatrix} I & G_0 \end{bmatrix} \right\|_\infty \leq \frac{1}{\varepsilon}. \quad (2)$$

Figure 6. System representation: (a) The close-loop system and (b) Rearrangement.

Hence, we can define the stability margin as follows:

Definition 1: Stability Margin[21]:

$$b(G_0, K) = \left(\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G_0 K)^{-1} \begin{bmatrix} I & G_0 \end{bmatrix} \right\|_{\infty} \right)^{-1} \quad (3)$$

Therefore, the closed-loop system is internally stable for all perturbations $\Delta = \begin{bmatrix} \Delta_N & \Delta_M \end{bmatrix}$ with $\|\Delta\|_{\infty} < \varepsilon$ if and only if $b(G_0, K) \geq \varepsilon$. That is, the designed controller K can guarantee internal stability for the PZT stage if the stability margin $b(G_0, K)$ is greater than the system uncertainties.

Table 2 and Figure 4 show that the transfer functions are varied at each experiment and, hence, we need to select the nominal plants for control design. Because the coprime factorization of a transfer function is not unique, we define the *gap* between a nominal plant G_0 and a perturbed plant G_{Δ} as follows:

Definition 2: Gap Metric [11]:

The smallest value of $\left\| \begin{bmatrix} \Delta_N & \Delta_M \end{bmatrix} \right\|_{\infty}$ which perturbs G_0 into G_{Δ} is called the *gap* between G_0 and G_{Δ} , and is denoted as $\delta(G_0, G_{\Delta})$.

Therefore, we should choose the nominal plants by minimizing the maximum gaps between the nominal plant and the perturbed plants, as in the following:

$$G_0 = \operatorname{argmin}_{G_0} \max_{G_i} \delta(G_0, G_i), \quad \forall G_i. \quad (4)$$

From Table 2, we selected the following nominal plants for the X-axis and Y-axis:

$$G_X = G_X^4 = \frac{48.63s^3 + 7.194 \times 10^4 s^2 + 8.517 \times 10^7 s + 3.024 \times 10^9}{s^4 + 3494 s^3 + 9.771 \times 10^6 s^2 + 2.648 \times 10^{10} s + 7.443 \times 10^{11}}, \quad (5)$$

$$G_Y = G_Y^9 = \frac{-112.8 s^3 + 1.123 \times 10^6 s^2 + 2.147 \times 10^9 s + 1.622 \times 10^{10}}{s^4 + 1847 s^3 + 8.254 \times 10^6 s^2 + 7.714 \times 10^9 s + 1.614 \times 10^{10}}. \quad (6)$$

These give the system gaps as $\delta(G_X, G_X^i) \leq 0.0953$ and $\delta(G_Y, G_Y^i) \leq 0.0437$.

Referring to Figure 7, the following loop-shaping techniques [9, 10] are usually applied for improving system performance:

- Multiply the nominal plant G by a pre-compensator W_1 and a post-compensator W_2 to form a shaped plant $G_s = W_2 G W_1$.
- Define the maximum stability margin $b_{\max}$ as:

$$b_{\max}(G_s, K) = \left(\inf_{K \text{ stabilizing}} \left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - G_s K)^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_{\infty} \right)^{-1}. \quad (7)$$

If $b_{\max}(G_s, K) \ll 1$, return to step (a) and adjust W_1 and W_2 . Finally, select an $\varepsilon \leq b_{\max}(G_s, K)$ to synthesize a stabilizing controller K_{∞} , such that

$$\left\| \begin{bmatrix} K_{\infty} \\ I \end{bmatrix} (I - G_s K_{\infty})^{-1} \begin{bmatrix} I & G_s \end{bmatrix} \right\|_{\infty}^{-1} \geq \varepsilon. \quad (8)$$

- Multiply K_{∞} by the weight functions, and implement $K = W_1 K_{\infty} W_2$ to control the PZT system G .

Figure 7. Loop shaping design.

In [10], we applied root-locus techniques to select weighting functions. The system achieved zero steady-state error with satisfactory settling time. However, the experimental responses looked very oscillatory because of system noise. Therefore, in [9], we applied frequency-based design to suppress this noise. In this paper, we will further discuss how the weighting functions influence the system performance.

The basic concept of selecting weighting functions is to let the loop transfer function $L = GK$ have high gain at low frequency ranges for tracking references, and have low gain at high frequency ranges for suppressing system noises. Based on this concept, we considered the following three weighting functions for robust control design. First, we considered a first-order integral weighting to eliminate steady-state error:

$$W_{1,x}^1 = W_{1,y}^1 = W_1^1 = \frac{10^5}{s}, W_{2,x}^1 = W_{2,y}^1 = W_2^1 = 1, \quad (9)$$

where the superscript 1 indicates that the weighting is first-order. This weighting uses an integral to eliminate the steady-state errors, but the system noise might not be effectively restrained because the PZT systems (see Figure 4) have a resonance at about 1000 rad/sec and the magnitude of the loop transfer function L is large with this weighting (see Figure 8). Therefore, we considered the following second-order weighting:

$$W_{1,x}^2 = W_{1,y}^2 = W_1^2 = \frac{6 \cdot 10^7}{s(s+900)}, W_{2,x}^2 = W_{2,y}^2 = W_2^2 = 1. \quad (10)$$

The loop transfer function L with this weighting is illustrated in Figure 8, where the magnitude is reduced so that we could expect smoother system responses. Lastly, we considered the following fourth-order weightings for further noise suppression:

$$W_{1,x}^4 = \frac{8 \cdot 10^3 (s+4000)^3}{s(s+900)^3}, \quad W_{1,y}^4 = \frac{2 \cdot 10^4 \times (s+4000)^3}{s(s+1100)^3}, \quad (11)$$

$$W_{2,x}^4 = W_{2,y}^4 = 1,$$

which can further reduce the magnitudes of the loop transfer function L , as shown in Figure 8.

We applied these weighting functions and designed the corresponding robust controllers listed in Table 3, where the stability margins are all greater than the system gaps. Therefore, these controllers can guarantee stability of a system with uncertainties. The designed controllers were implemented for performance verification in Section 4.

Experiments

In this section, we implemented the designed controllers on the PZT stage and verified the performance through experiments. The inputs were set as steps with magnitudes of 200, 500, and 1000nm; the system

responses with these controllers are illustrated in Figure 9. We compared the system performance by settling time, overshoot, maximum absolute error (MAE), and root-mean-square error (RMSE), as listed in Table 4. First, the settling time was improved with higher-order weightings because they provided higher system bandwidth (see Figure 4). Second, the overshoots and MAE were also reduced by the fourth-order weighting. Lastly, the RMSE was also reduced with a higher-order weighting because the magnitude of the loop-transfer function L was reduced (see Figure 4). Based on the results, the controllers K_x^4 and K_y^4 using the fourth-order weightings provided a settling time of less than 0.013s, an overshoot of less than 1.2%, a MAE of less than 11.3nm, and a RMSE of less than 1nm, which were better than the lower-order weightings.

To appreciate the merits of the aforementioned robust control design procedures, in Figure 10, we compared the system performance with the following traditional PID control:

$$K_x^{PID} = \frac{9.984 \cdot 10^6 (s+382)}{s(s+299)}, \quad (12)$$

$$K_y^{PID} = \frac{1.003 \cdot 10^7 (s+401)}{s(s+317)}. \quad (13)$$

Figure 8. Weighted plants: (a) for G_x^0 and (b) for G_y^0 .

Figure 9. Step responses: (a) X-axis: 200nm, (b) Y-axis: 200nm, (c) X-axis: 500nm, (d) Y-axis: 500nm, (e) X-axis: 1000nm and (f) Y-axis: 1000nm.

and root-locus-based control of [11]:

$$\begin{aligned}
 K_X^R &= \frac{1.435s^5 + 2650s^4 + 3.609 \times 10^6 s^3 + 1.936 \times 10^9 s^2 + 3.141 \times 10^{11} s + 1.93 \times 10^{12}}{s^5 + 2263s^4 + 3.174 \times 10^6 s^3 + 1.982 \times 10^9 s^2 + 4.376 \times 10^{11} s + 2.77 \times 10^{12}}, \\
 W_X^R &= \frac{3 \times 10^6 (s + 600)}{s(s + 250)}, \\
 K_Y^R &= \frac{1.491s^4 + 3100s^3 + 3.296 \times 10^6 s^2 + 8.706 \times 10^8 s + 2.738 \times 10^{10}}{s^4 + 3041s^3 + 2.928 \times 10^6 s^2 + 1.225 \times 10^9 s + 4.084 \times 10^{10}}, \\
 W_Y^R &= \frac{10^{10} (s + 400)}{s(s + 300)}.
 \end{aligned}$$

14)

We selected the PID parameters according to the loop-shaping principles. That is, we chose suitable PID parameters such that the loop transfer function $L=GK$ have high gain at low frequency ranges for tracking references, and have low gain at high frequency ranges for suppressing system noises. We then calculated the phase margins and modified the PID parameters iteratively. The responses with the PID control and the root-locus-based control were very oscillatory because the magnitudes of their loop transfer function were large at the concerned noise frequencies.

Summary

This paper has demonstrated robust loop-shaping control for a piezoelectric nano-positioning stage, and discussed the impacts of weighting functions on system performance. We used Bode plots to investigate the relation between weighting functions and their ability to suppress system noises. The designed robust controllers were implemented for experimental verification and achieved a RMSE of less than 1nm. Similar procedures can be applied to design robust loop-shaping controllers for general systems.

Figure 10. Performance comparison of three controllers: (a) X-axis and (b) Y-axis.

Reference

- [1] Y. S. Kung and R. F. Fung, "Precision control of a piezoceramic actuator using neural networks," *J. Dyn. Sys., Meas., Control*, vol. 126, no. 1, pp. 235-238, 2004.
doi: [10.1115/1.1651535](https://doi.org/10.1115/1.1651535)
- [2] G. Song, Z. Jinqiang, Z. Xiaoqin, and J. A. De Abreu-Garcia, "Tracking control of a piezoceramic actuator with hysteresis compensation using inverse preisach model," *IEEE/ASME Transactions on Mechatronics*, vol. 10, no. 2, pp. 198-209, 2005.
doi: [10.1109/TMECH.2005.844708](https://doi.org/10.1109/TMECH.2005.844708)
- [3] C. H. Liu, W. Y. Jywe, Y. R. Jeng, T. H. Hsu, and Y. T. Li, "Design and control of a long-traveling nano-positioning stage," *Precision Engineering*, vol. 34, no. 3, pp. 497-506, 2010.
doi: [10.1016/j.precisioneng.2010.01.003](https://doi.org/10.1016/j.precisioneng.2010.01.003)
- [4] M. Brokate and J. Sprekels, *Hysteresis and phase transitions*. New York: Springer, 1996.
- [5] A. Visintin, *Differential models of hysteresis*. Berlin; New York: Springer, 1994.
- [6] Y. K. Wen, "Method for random vibration of hysteretic systems," *Journal of the Engineering Mechanics Division*, vol. 102, no. 2, pp. 249-263, 1976.
- [7] D. Song and C. J. Li, "Modeling of piezo actuator's nonlinear and frequency dependent dynamics," *Mechatronics*, vol. 9, no. 4, pp. 391-410, 1999.
doi: [10.1016/S0957-4158\(99\)00005-7](https://doi.org/10.1016/S0957-4158(99)00005-7)
- [8] P. Mercorelli, N. Werner, U. Becker, and H. Harndorf, "A robust model predictive control using a feedforward structure for a hybrid hydraulic piezo actuator in camless internal combustion engines," in *2012 9th International Multi-Conference on Systems, Signals and Devices (SSD)*, 2012, pp. 1-6.
doi: [10.1109/SSD.2012.6198023](https://doi.org/10.1109/SSD.2012.6198023)
- [9] S. Yingfeng and K. K. Leang, "Repetitive control with prandtl-ishlinskii hysteresis inverse for piezo-based nanopositioning," in *American Control Conference (ACC)*, 2009, pp. 301-306.
doi: [10.1109/ACC.2009.5160618](https://doi.org/10.1109/ACC.2009.5160618)
- [10] M. Steinbuch, S. Weiland, and T. Singh, "Design of noise and period-time robust high order repetitive control, with application to optical storage," *Automatica*, vol. 43, no. 12, pp. 2086-2095, 2007.
- [11] V. Sethi, M. A. Franchek, and G. Song, "Active multimodal vibration suppression of a flexible structure with piezoceramic sensor and actuator by using loop shaping," *Journal of Vibration and Control*, vol. 17, no. 13, pp. 1994-2006, 2011.
doi: [10.1177/1077546310393440](https://doi.org/10.1177/1077546310393440)
- [12] F. C. Wang, L. S. Chen, Y. C. Tsai, C. H. Hsieh, and J. Y. Yen, "Robust loop shaping control for a nano-positioning stage," *Journal of Vibration and Control*, to appear, 2013.
- [13] F. C. Wang, Y. C. Tsai, C. H. Hsieh, L. S. Chen, and C. H. Yu, "Robust control of a two-axis piezoelectric nano-positioning stage," in *the 18th IFAC World Congress*, Milano, Italy, 2011, vol. 18, pp. 35939-33544.
doi: [10.3182/20110828-6-IT-1002.00741](https://doi.org/10.3182/20110828-6-IT-1002.00741)
- [14] T. T. Georgiou and M. C. Smith, "Optimal robustness in the gap metric," *IEEE Transactions on Automatic Control*, vol. 35, no. 6, pp. 673-686, 1990.
doi: [10.1109/9.53546](https://doi.org/10.1109/9.53546)
- [15] *Mercury ii 5800Si*, [Online]. Available: <http://www.microsys.com/specifications/high-performance-encoders/mercury-ii-5800Si>
- [16] *Ni pxi-8108*, [Online]. Available: <http://sine.ni.com/nips/cds/view/p/lang/zht/nid/206087>

- [17] "E-663 three-channel piezo driver," Available: <http://www.physikinstrumente.com/en/products/prdetail.php?sortnr=601800>
- [18] J. Chow, A. Ebeling, and B. Teskey, "Low cost artificial planar target measurement techniques for terrestrial laser scanning," presented at the FIG Congress 2010: Facing the Challenges - Building the Capacity, Sydney, Australia, 2010.
- [19] K. Zhou, J. C. Doyle, and K. Glover, *Robust and optimal control*. Upper Saddle River, N.J.: Prentice Hall, 1996.
- [20] K. Zhou and J. C. Doyle, *Essentials of robust control*. Upper Saddle River, N.J.: Prentice Hall, 1998.
- [21] P. van Overschee and B. L. de Moor, *Subspace identification for linear systems : Theory, implementation, applications*. Boston: Kluwer Academic Publishers, 1996.

Table 3. The designed robust controllers.

Transfer function	Gap
$K_x^1 = \frac{1.005s^4 + 2699s^3 + 2.899 \times 10^6 s^2 + 1.449 \times 10^{10} s + 5.043 \times 10^{11}}{s^4 + 6242s^3 + 9.032 \times 10^6 s^2 + 1.419 \times 10^{10} s + 5.066 \times 10^{11}}$	0.656
$K_y^1 = \frac{2.209 s^4 + 2841s^3 + 1.44 \times 10^6 s^2 + 5.94 \times 10^8 s + 1.021 \times 10^{10}}{s^4 + 4452s^3 + 2.334 \times 10^6 s^2 + 1.295 \times 10^9 s + 2.256 \times 10^{10}}$	0.4124
$K_x^2 = \frac{1.117s^5 + 4862s^4 + 1.415 \times 10^7 s^3 + 3.91 \times 10^{10} s^2 + 2.8 \times 10^{13} s + 9.18 \times 10^{14}}{s^5 + 4931s^4 + 1.535 \times 10^7 s^3 + 3.524 \times 10^{10} s^2 + 2.938 \times 10^{13} s + 1.026 \times 10^{15}}$	0.6269
$K_y^2 = \frac{1.372s^5 + 2212s^4 + 1.441 \times 10^6 s^3 + 7.565 \times 10^8 s^2 + 2.337 \times 10^{11} s + 3.839 \times 10^{12}}{s^5 + 2044 s^4 + 1.84 \times 10^6 s^3 + 8.85 \times 10^8 s^2 + 3.083 \times 10^{11} s + 5.266 \times 10^{12}}$	0.4891
$K_x^4 = \frac{1.337 s^7 + 8315s^6 + 2.912 \times 10^7 s^5 + 8.367 \times 10^{10} s^4 + 1.337 \times 10^{14} s^3 + 1.01 \times 10^{17} s^2 + 2.986 \times 10^{19} s + 9.142 \times 10^{20}}{s^7 + 6554s^6 + 2.384 \times 10^7 s^5 + 6.907 \times 10^{10} s^4 + 1.163 \times 10^{14} s^3 + 9.881 \times 10^{16} s^2 + 3.691 \times 10^{19} s + 1.222 \times 10^{21}}$	0.5990
$K_y^4 = \frac{3.761s^7 + 1.768 \times 10^4 s^6 + 4.207 \times 10^7 s^5 + 7.41 \times 10^{10} s^4 + 9.35 \times 10^{13} s^3 + 6.857 \times 10^{16} s^2 + 2.163 \times 10^{19} s + 3.935 \times 10^{20}}{s^7 + 7760s^6 + 3.025 \times 10^7 s^5 + 7.745 \times 10^{10} s^4 + 1.359 \times 10^{14} s^3 + 1.503 \times 10^{17} s^2 + 7.881 \times 10^{19} s + 1.48 \times 10^{21}}$	0.2991

Table 4. Statistic data of Figure 9.

Experiment		$W_x^1 G_x / W_y^1 G_y$	$W_x^2 G_x / W_y^2 G_y$	$W_x^4 G_x / W_y^4 G_y$
200nm	Settling time (s)	0.021/0.048	0.014/0.020	0.012/0.013
	Overshoot (%)	0.000/0.000	0.000/0.000	0.000/0.000
	MAE (nm)	0.940/1.900	1.100/1.650	0.855/1.605
	RMSE (nm)	0.421/0.621	0.447/0.528	0.426/ 0.399
500nm	Settling time (s)	0.018/0.045	0.012/0.019	0.011/0.012
	Overshoot (%)	0.000/0.000	2.089/0.000	0.000/0.000
	MAE (nm)	1.102/2.643	5.052/3.851	0.924/2.035
	RMSE (nm)	0.588/0.582	0.456/ 0.621	0.433/0.673
1000nm	Settling time (s)	0.013/0.042	0.011/0.017	0.010/0.012
	Overshoot (%)	0.000/0.000	3.641 /1.322	1.102/0.450
	MAE (nm)	1.651/2.841	33.41 /13.21	11.24/ 5.894
	RMSE (nm)	0.612/0.766	0.600/0.684	0.477/0.495