Hydraulic Servo System Control Using Differential Evolution Based Robust Structure Specified $H_\infty$ Controller

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Abstract: Synthesis of an $H_\infty$ controller usually produces a very high order of controller, and one which is much higher than the plant, causing difficulty in implementing the controller in practical applications, especially those using small microcontrollers. However, systems are always subject to disturbances and un-modeled dynamics. This research introduces modeling and simulations of lower order structures, specifically an $H_\infty$ robust controller, to control a hydraulic servo system. A low order controller shortens the gap between the complicated $H_\infty$ controllers to the practical embedded control system application. Simulation results show that the proposed controller gives satisfactory results with small settling times and no steady state error. The resulting controller also produced better responses than that of a full order $H_\infty$ controller generated using the Matlab Robust Control Toolbox.

Keywords: Servo Hydraulic; Closed Loop Control; H-Infinity Control

Introduction

A servo system powered by hydraulic actuator is called a hydraulic servo system. Hydraulic position servo systems are widely used in many applications including industrial production, construction, military, and transportation. However, controlling a hydraulic servo system raises certain challenges including non-linearity, un-modeled dynamics, and uncertainties due to variations in fluid volumes and leakage. Tadese et al. [1] modeled and simulated a fuzzy-PID position controller for a hydraulic servo system. Skarpetis et al. [2] proposed robust position tracking for a hydraulic servo system, using the Internal Model Principle modified with a Hurwitz invariability technique and a Simulated Annealing Algorithm. A loop shaping based robust controller for hydraulic servo system was also presented by Zhang et al. [3]. Robust optimization in an $H_\infty$ control has been extensive studied in the past few years [4,5,6,7]. Basically it can be solved in the frequency domain [4,7] or time domain [6]. However, both methods produce controllers that have a much higher order of the controller than the plant itself, which makes the implementation of an $H_\infty$ robust controller far from practical. This research proposes a lower order robust structure specified $H-$ controller based on parameter optimization to obtain desired $H-$ performance. To achieve optimal parameters, Differential Evolution was used to prevent the search from being trapped in a local optimum. Differential Evolution is an evolution based optimization [8]. Sutyasadi [9] showed that a DE-based $H_\infty$ controller outperforms a PID controller given uncertainties. The remainder of this paper is organized as follows. Section II presents the modeling of the hydraulic servo system. The $H-$ controller and n-modeled dynamics and uncertainties of the system are explained in section III. Section IV presents simulation results for the proposed controller responses under uncertainties. Section V synthesizes a full order of the $H-$ controller using the robust control Matlab Toolbox and compares it to the proposed controller. Conclusions are given in section VI.
Hydraulic Model

This section provides a derivation of the hydraulic servo system. The hydraulic servo system consists of hydraulic actuators, electronic drives, and a position transducer [13,14]. The mathematical model of the system describes the relationship between displacement output of the load and voltage input to the solenoid that moves the spool. Figure 1 shows a hydraulic actuator with a four-way valve configuration.

![Figure 1. Hydraulic actuator with four-way valve configuration.](image)

The development objective for the actuator system dynamics is a strict feedback control with a fixed boundary layer to obtain precise position control of a nonlinear electro-hydraulic servo system [15]. To represent the servo valve dynamics through a wider frequency range, the transfer function is used as an approximation of the valve dynamics.

The data is separated into estimation data, which is used to identify unknown system parameters and measurement data. To excite all the relevant frequencies of the systems and to construct a good model, the frequencies are set to the sinusoidal inputs with a range of 1 to 6 Hz and pressure of 5 Mpa. In the conventional design of a hydraulic servo system, third order transfer function is generally used, as given in (1) below

\[
G(s) = \frac{K_q \omega_n^2}{s(s^2 + 2\delta\omega_n s + \omega_n^2)}
\]

(1)

Where \(K_q\) is the flow constant gain, \(A_1\) is the actuator ramp area, \(\omega_n\) is Natural Frequency, and \(\delta\) is the damping ratio.

In the frequency response analysis, we measure the amplitude of oscillations at the signal frequency. We initially carried out a set of experiments using the open loop system to determine the amplitude of oscillations, which occur at the signal frequency. To observe the signal frequency component of the response alone, experiments were carried out using signal frequencies of 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5 and 6 Hz at 5 Mpa pressure. Overall, the system identification is done by fitting a third order polynomial. The system transfer function is found to be:

\[
G(s) = \frac{9272}{s^3 + 205.5s^2 + 10560s}
\]

(2)

From Fig. 2, the correspondence levels between the predicted model and the experimental data for the second order and third order models are respectively about 74.16% and 80.77%. In this experiment, the input test signal from 1 to 6 Hz is applied as the estimation data. The parameter values are then varied by optimization until best fitting is achieved. The Matlab identification toolbox software is used to create the system’s mathematical model. The process-modeling tool is selected to customize the structure of the identified model based on the knowledge of the second order and third order of the hydraulic plant. Finally, Fig. 2 compares the experimental outputs and the predicted model using estimation data.

Structure Specified H\(\infty\) Robust Controller

System uncertainties

Multiplicative uncertainties in the system are shown in Fig. 3. \(G_n(s)\) is the nominal system, \(\Delta G(s)\) is the system perturbation, \(K(s)\) is the controller, \(r(t)\) is the reference input, \(e(t)\) is the tracking error, \(d(t)\) is the external disturbance, and \(y(t)\) is the system output.

The perturbed system is expressed by
\[ G(s) = G_n(s)(1 + \Delta(s)) \]  
(3)

Thus, the multiplicative system perturbation is determined from

\[ \Delta(s) = \left( \frac{\bar{G}(s)}{G_n(s)} - 1 \right) \]  
(4)

The nominal transfer function of the hydraulic servo system is shown in (1). Equation (2) can be re-written as:

\[ G(s) = \frac{0.878\omega_n^2}{s^3 + 2\delta\omega_n s + \omega_n^2} \]  
(5)

With \( \delta = 0.999 \) and \( \omega_n = 102.76 \) in the nominal plant. Uncertainties were included as the variation of \( \omega_n \) and \( \delta \). Figure 4 shows the singular value of the uncertainties.

![Figure 4. Single value of uncertainties.](image)

**Mixed sensitivity H\(\infty\) control**

If a controller \( K(s) \) is achieved with a stable closed loop system, then robust stable performance will follow the inequality:

\[ J_{s,b} = \left\| W_s T(s) \right\|_{\infty} < 1 \]  
(6)

and robust stability against system perturbation will follow the inequality:

\[ J_{s,\Delta} = \left\| W_s T(s) \Delta(s) \right\|_{s} < 1 \]  
(7)

where \( S(s) \) and \( T(s) \) are respectively the sensitivity and complementary sensitivity function. \( W_s \) is the sensitivity weight that attenuates the external disturbance and \( W_T \) is the complementary sensitivity that upper bounds the multiplicative perturbation. The control block diagram is shown in Fig. 5.

![Figure 5. Mixed sensitivity control block diagram.](image)

Following Skogestad’s method [9], the sensitivity weight is:

\[ W_s(s) = \frac{0.5s + 1}{s + 0.001} \]  
(8)

The weight to the bounded uncertainties was set using Matlab and was designed so that the weight upper bounded the uncertainties in all frequency ranges. The complementary sensitivity is set as:

\[ W_t(s) = \frac{1.148s^2 + 78.84s + 858.8}{s^2 + 118.4s + 5378} \]  
(9)

Figure 6 shows the uncertainties with the weight.

![Figure 6. Singular value of the uncertainties and the complementary sensitivity weight.](image)

**Differential Evolution Optimization**

Given the nominal plant transfer function, sensitivity function, complementary sensitivity function, and the structure of the controller, the parameters of the controller can be achieved using Differential Evolution (DE). DE is a new heuristic approach to minimize nonlinear and non-differentiable functions [10]. DE search is
conducted in parallel and begins with a random population. Through mutation, recombination, and selection, it retains only good individuals [11].

The DE parameters set as follows: population = 50, differential weight = 0.8, and crossover probability = 0.7. Figures 7 to 9 show the searching of controller parameter during the evaluation process.

The structure specified a mixed sensitivity $H_{\infty}$ controller for which the parameter derived using DE [12] is:

$$K(s) = 79.4786 + \frac{33.0537}{s} + 2.6622s$$  \hspace{1cm} (9)

### Hardware System Architecture

The HSS must follow the control theory guidelines to improve the piston velocity in the HSS. The HSS hardware system is divided into two parts, which are explained below.

#### Hardware Design

First, we construct the mechanical model of an electro-hydraulic system. The simulated response of the model provides insight into the behavior of the electrohydraulic system.

As shown in Fig. 10, (1) is the linear potentiometer; (2) is the double cylinder; (3) is the servo valve; (4) is the pressure relief valve representing fluid flows in and out of the valve; (5) the pressure unit is the input and output line pressures and (6) is the system microcontroller. (A) is a supply flow port, (B) is the return flow port, (F) is Force, (P) is Pressure, and (T) is Tank.

The HSS consists of a hydraulic pump, servo valve, actuator, transducer, power supply, and microcontroller. The hydraulic system model is shown in Fig.11.
A microcontroller (PIC 18F458)-based control system was developed to control the hydraulic servo system, in conjunction with the data acquisition processor. Figure 12 shows a schematic of the microcontroller system design. The mass flow rate across the five-port valve is controlled by manipulating the spool offset, by controlling the current supplied to the solenoid.

**Simulation Result**

Figure 13 shows the simulation result of the proposed controller under uncertainties. For the nominal plant without uncertainty, the system response has oscillation but no overshoot. Settling times in nominal mode are small, almost 0.1 second. However, under uncertainties, some responses in some conditions have overshoot, though still less than 20%. Settling times around 0.05 to 0.1 for any possibility among the uncertainties are rather small.

**Full Order H∞ Robust Controller**

Through its robust control toolbox, Matlab provides a method to synthesize a mixed sensitivity robust controller. To validate the proposed controller, a high or full order mixed sensitivity $H_\infty$ robust controller is derived. The resulting controller is (10).

The controller is simulated using the same plant and the same range of uncertainties. Figure 15 shows system responses of the system under uncertainty, where all system responses are stable without overshoot or oscillation. However, settling times of all the responses are rather long.
The structure than the conventional H\(\infty\) controller, thus allowing for embedded controller implementations. The proposed controller has overshoot and oscillation under some conditions, but the settling time is much smaller than that of the full order controller. Maximum overshoot was 18%. Response with the highest overshoot has the smallest settling time. Overshoot did not occur in all uncertainty conditions. Settling times vary between 0.05 and 0.1 second. The conventional high or full order controller gave a better response in term of overshoot and oscillation. However, the smallest and largest settling times were respectively 1.65 and nearly 2 seconds, which is much bigger than the proposed controller settling times.

Overall, it can be concluded that the structure specific mixed sensitivity H\(\infty\) robust controller has satisfactory performance. Moreover, its structure in the form of a PID controller provides significant benefits due to its popularity in industrial applications. The structure also allows for the controller to be programmed into a small microcontroller, while still providing robust performance.

\[
K(s) = \frac{3.431e07s^5 + 1.111e10s^4 + 1.382e12s^3 + 8.081e13s^2 + 1.948e15s + 3.742e06}{s^6 + 1.071e04s^5 + 5.278e07s^4 + 1.469e11s^3 + 1.771e13s^2 + 8.118e14s + 8.118e11}
\]

The singular value plot of the sensitivity, complimentary sensitivity, and their weights that all the requirements are fulfilled. Figure 16 shows that the singular plot of the sensitivity, complimentary sensitivity, and its weights also satisfy the requirements.

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