Precision Positioning with Shape-Memory-Alloy Actuators

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Abstract: The primary purpose of this study is to improve the positioning performance of shape-memory-alloy (SMA) actuators. In order to achieve this goal, the system nonlinearity was reduced with the inversion of a nonlinear model. The system could then be approximated with a linear model. It is easy to construct the corresponding model-reference-adaptive-system (MRAS) based on this linear model. Experimental results show that the MRAS is robust with respect to external disturbances and improves the positioning performance. In addition, with the proposed control scheme, the simulation results will closely match experimental results, which is useful to predict the system performance at the controller-design stage.

Keywords: Adaptive control; hysteresis compensation; model reference adaptive system; precision positioning; shape memory alloy

1. Introduction

Shape Memory Alloys (SMAs) have the ability to recover a predetermined shape through the application of heat, even after a large strain occurs [1]. In addition, SMAs have been used broadly in different applications due to many features such as durability, low noise during operation, a simple mechanism for energy transformation, high power-to-weight ratio, and being innocuous for living bodies [1-4]. For example, in aeronautic engineering SMAs can be used to control the opening area of an exhaust nozzle on a gas turbine [5], reducing the system weight since they have a high power-to-weight ratio compared to conventional actuators. In civil engineering, SMAs can be integrated with other passive, semi-active, or active components, in order to reduce damage caused by environmental impact due to their high damping capacity, durability, and fatigue resistance [6]. In the case of bioengineering, SMAs have been used in prostheses [7-9] and surgeries [4, 10] due to their lightweight, dexterous, and biologically innocuous properties.

Although SMAs have many superior features in comparison to other materials or actuators, they are often difficult to control due to high nonlinearity and sensitivity to variations of the ambient temperature [11]. Therefore, in order to enhance the performance of SMAs, the development of good models and the application of suitable control schemes are two issues to be studied.

In order to describe the characteristics of SMAs, a variety of constitutive models have been developed [12, 13]. These constitutive models feature physical insights of SMAs since each parameter has a physical meaning. However, they often consist of complicated mathematical equations and may require many experiments to determine each parameter. In contrast to modeling, many control schemes were studied and applied to control SMAs to improve the performance. In the cases of simple adaptive control (SAC) and sliding mode control for instance, they increase the system robustness against modeling uncertainties and external disturbances [14-16]. Other control schemes, for example a PID (proportional-integral-derivative) controller combined with fuzzy logic [17] and neural-network-based controllers [18] were utilized to
cope with the nonlinearity of SMAs; all of them provided good choices to control SMAs.

In this article, the nonlinearity of SMAs is modeled with the Preisach model [19, 20] instead of constitutive models. Although the Preisach model is constructed by fitting experimental hysteresis curves which do not directly relate to material properties (i.e. the Preisach model provides fewer insights into the mechanism controlling the material transformation), it still reflects the hysteresis behavior and is convenient for simulation and control applications [21-23]. Therefore, the nonlinearity of SMAs was initially reduced using the inverse-Preisach-model-based control [24, 25], allowing the new system to be approximated with a linear model.

It is simple to construct the corresponding model-reference-adaptive-system (MRAS) [26] based on this linear model to cope with modeling uncertainties and external disturbances. In addition, it is expected that the simulation will closely match the experimental results since the inverse-Preisach-model-based control will transform the SMAs system to approach a linear system used for the controller-design stage.

The remainder of this article is organized as follows. The material and method used in this research are covered in Sec. 2. The experimental results and discussions are given in Sec. 3. Conclusions are presented in Sec. 4.

2. Material and Method

2.1. Experiment Setup

Figure 1 shows the schematic diagram of the experimental setup. A nickel-titanium SMA-wire (diameter $\phi=150\mu$m; typical contraction time $T_c=1s$; typical relaxation time $T_d=2s$; manufactured by Dynalloy Inc.) fixed to the SMA-holder with pre-tension $F_t$ of 3N (measured with a load cell, SMT S-type, Interface Inc.) was constructed. Varying the applied voltage on the SMA-wire, allows its length to change and be recorded with a linear-variable-differential transformer (LVDT) (JEC-AG DC-DC, Honeywell Inc.).

![Figure 1. Schematic diagram of the experimental setup. (a) Is the top view and (b) is the side view.](image)

Remark 1. In this research, the maximum deformation ratio and the maximum applied voltage for the SMA are limited to 4% (i.e., 4mm and 4V, respectively. This is based on the recommended operating conditions provided by the manufacturer.

2.2. Control Schemes

As shown in Figure 2(a), the SMA-wire was modeled as two cascaded sub-systems: hysteresis $H$ and linear dynamics $G$. The control strategy of this research is to (i) utilize an inverse hysteresis model $H_0^{-1}$ to reduce the system’s nonlinearity in order to model it as a linear model $G_0$, and (ii) construct an MRAS based on a reference model $G_m$ such that the adaptive controller $C_{adp}$ can make the deformation of SMA $d^{SMAd}_{mv}$ equal the output of the reference model $d_m$ (see Figure 2(b)). The detailed procedures are elaborated in the following two subsections.

2.2.1. Preisach-model-based control

In this research, the nonlinear hysteresis is modeled using the Preisach model [19] and the deformation of the SMA $d^{SMAd}_{mv}(t)$ can be described using the integral of all weighted Preisach hysterons,
\[ d_{\text{SMA}}(t) = \int_{\alpha, \beta} \mu(\alpha, \beta) \gamma_{\alpha \beta}(u(t)) d\alpha d\beta, \]

where \( \mu(\alpha, \beta) \) is a weighting function in the Preisach model, and \( \gamma_{\alpha \beta}(u(t)) \) is the Preisach hysteron whose on-off state is determined by the input voltage \( u(t) \).

In order to identify the weighting function \( \mu(\alpha, \beta) \), the Preisach plane needs to be considered in a discrete form [27], i.e., Eq. (1) needs to be expressed as,

\[ d_{\text{SMA}}(t) = \sum_{k=1}^{n} \gamma_k \mu_k A_k, \]

where \( \gamma_k \) is the discrete Preisach hysteron which has a value of "1" for the on-state or "0" for the off-state, \( A_k \) represents the \( k^{th} \) area in the discretized Preisach plane, and \( \mu_k \) is the weighting function for area \( A_k \).

As an example, Figure 3, shows a four interval case (i.e. \( N = 4 \)) where the total number, \( n \), of the discrete areas in the Preisach plane is ten (i.e., \( n = N(N+1)/2 \)), and the corresponding deformation at time \( t_1, t_2, \) and \( t_3 \) are,

\[ d_{\text{SMA}}(t_1) = \sum_{i=1}^{10} \mu_i A_i, \]
\[ d_{\text{SMA}}(t_2) = \sum_{i=1}^{10} \mu_i A_i + \sum_{i=5}^{10} \mu_i A_i, \]
\[ d_{\text{SMA}}(t_3) = \sum_{i=1}^{3} \mu_i A_i + \sum_{i=5}^{6} \mu_i A_i + \sum_{i=9}^{10} \mu_i A_i. \]

Therefore, by applying a test input \( u \) (the input shown in Figure 3(d)) and mapping the input-to-output relation experimentally as,

\[ [d_{\text{SMA}}] \approx [\Psi][\eta], \]

the weighting function \( \mu_k \) can be obtained from \( [\eta] \) by solving Eq. (4) using the least-squares method [25], where \( [d_{\text{SMA}}] \) is the vector of the measured output (i.e., SMA deformation), \( [\Psi] \) is the vector of the weighting function \( \mu_k \) multiplied by the known area \( A_k \), and \( [\eta] \) is the hysterons matrix consisting of the on-state and off-state (i.e., 1 or 0). The hysteresis model \( H_1 \) constructed is shown in Figure 4(a).

Similarly, the inverse hysteresis can be modeled using the same concept since the inverse hysteresis curve and the hysteresis curve are symmetric with respect to \( d_{\text{SMA}} = u \). In other words, the inverse hysteresis can be considered as a kind of hysteretic effect which exchanges the input with output. Therefore, rather than conducting any experiment, the inverse hysteresis model \( H_0 \) can be constructed through interpolation of the input-to-output relation of \( H_0 \) and deriving the "new" weighting function \( \mu^r \) (the result is shown in Figure 4(b)).

2.2.2. Adaptive control

In this research, adaptive control based on MRAS is used to compensate for the modeling uncertainties and external disturbances. In particular, the MIT rule [26] is
utilized and the MRAS shown in Figure 2(b) can be converted to the overall control block diagram of Figure 5. It should be noted that given a reference model $G_e$, the goal of the MIT rule is to make the system output $d_{SMA}$ equal the output of the reference model $d_m$. The process can be elaborated as follows.

The definition of a cost function $J(\theta)$ in terms of the error $e$ is

$$J(\theta) = \frac{1}{2} (e)^T (e) = \frac{1}{2} (d_{SMA} - d_m)^T (d_{SMA} - d_m),$$

(5)

which means the time derivative $J$ of is,

$$\dot{J} = \frac{dJ}{dt} = \frac{d}{dt} \frac{\partial J}{\partial \theta} = e \frac{\partial e}{\partial \theta},$$

(6)

where $\theta = [\theta_1, \theta_2]$ is the adjustable parameter. In order to make the error $e$ approach zero, the time derivative of $\theta$ can be designed with a positive constant $\lambda$

$$\dot{\theta} = -\lambda e \frac{\partial e}{\partial \theta},$$

(7)

such that Eq. (6) becomes,

$$\dot{J} = -\lambda \left( e \frac{\partial e}{\partial \theta} \right)^T \leq 0,$$

(8)

i.e., the cost function $J$ will decrease as long as error $e$ is non-zero.

Given the reference model $G_e = \frac{b_m}{s + am}$ and the nominal model $G_s = \frac{b}{s + a}$ as shown in Figure 5, the system output $d_{SMA}$ and the reference output $d_m$ can be expressed as,

$$\frac{d(d_{SMA})}{dt} = -a d_{SMA} + b u_{adp},$$

(9)

$$\frac{d(d_m)}{dt} = -a_m d_m + b_m d_s,$$

(10)

If the control law is set to be,

$$u_{adp} = \theta_1 d_s - \theta_2 d_{SMA},$$

(11)

Eq. (9) can be rewritten as,

$$\frac{d(d_{SMA})}{dt} = -(a + b \theta_2) d_{SMA} + b \theta_1 d_s,$$

(12)

Therefore, by inspecting Eq. (10) and Eq. (12), two equations will be identical by setting the adjustable parameter $\theta$ to be,

$$\theta_1 = \frac{b_m}{b} \triangleq \theta_{SMA},$$

(13)

$$\theta_2 = \frac{a_m}{b} \triangleq \theta_{ad},$$

(14)

which is called perfect model-following, where $\theta_{SMA}$ and $\theta_{ad}$ are the corresponding values.

Finally, the error sensitivity with respect to the adjustable parameter (defined as $\frac{\partial e}{\partial \theta}$ ) can be approximated with the perfect model-following condition,

$$\frac{\partial e}{\partial \theta_1} = \frac{b}{p + a + b \theta_2} d_s \approx \frac{b}{p + a_m} d_{SMA},$$

(15)

$$\frac{\partial e}{\partial \theta_2} = \frac{-b \theta_1}{(p + a + b \theta_2)^2} d_s \approx \frac{-b}{(p + a_m)} d_{SMA},$$

(16)

where $p = \frac{d}{dt}$. Substitution of Eqs. (15) and (16) into Eq. (7) gives the time derivative of the adjustable parameter

$$\frac{d\theta_1}{dt} = -\lambda e \frac{\partial e}{\partial \theta_1} = -\gamma \left( \frac{a_m}{p + a_m} d_{SMA} \right) e,$$

(17)

$$\frac{d\theta_2}{dt} = -\lambda e \frac{\partial e}{\partial \theta_2} = -\gamma \left( \frac{a_m}{p + a_m} d_{SMA} \right) e,$$

(18)

where $\gamma = \frac{\lambda b}{a_m}$ is the adaptation gain. This completes the control structure shown in Figure 5.

Remark 2. In practice, once the SMA system is linearized to $G_s$ (see Figure 5), any robust feedback controller (even a simple PID-controller) can be added to the control-loop to improve the performance. However, the MRAS-controller utilized in this research is designed in the Lyapunov-stability sense regardless of the modeling uncertainties and external disturbances. Therefore, it is expected to exhibit greater robustness. In the worst case, the model $G_s$ may deviate from the ideal system $G$ significantly and saturation on the input $u_{adp}$ may occur. The error $e$ will not vanish in this case.
3. Experimental results and discussions

3.1. Hysteresis compensation

The hysteresis of the system can be compensated with the inverse hysteresis model $H_b^{-1}$ and using the procedure shown in Sec. 2.2.1. In other words, given a desired deformation of SMA $d_a$, the input to compensate for the hysteresis can be calculated as,

$$u = H_b^{-1}(d_a). \quad (19)$$

As shown in Figure 6(b), before compensation, the SMA wire shows high nonlinearity; after compensation, the nonlinearity is reduced substantially.

![Figure 6. Hysteresis compensation. (a) The desired deformation of SMA $d_a$ in the time domain, and (b) comparison of the hysteric effect (dotted line: without compensation; dashed line: simulation result with inverse-hysteresis-model compensation; solid line: experimental result with inverse-hysteresis-model compensation).](image)

**Remark 3.** The quantization errors from discrete formulation of the hysteresis model can be analyzed by increasing the partition number $N$ until the quantization error levels off [28]. In this research, we set $N = 40$ to maintain accuracy and minimize computational expense.

3.2. Linear dynamics compensation

Since the system’s nonlinearity has been reduced with the inverse hysteresis model $H_b^{-1}$, the linear dynamic model $G_0$ can be obtained from the frequency response of a sine-sweep test. At first, a desired deformation of SMA $d_a$ in a sine-wave form is set as the input. The corresponding voltage to compensate for the hysteretic effect is then calculated via Eq.(19) and applied to the SMA to measure the deformation of SMA $d_{SM}$ (i.e., the output). Therefore, by constantly varying the frequency of the sine-wave and analyzing the corresponding input-output relation, the linear dynamic model $G_0$ can be obtained. It can be seen from Figure 7 that the measured system is similar to a first-order system with a bandwidth of around $0.1Hz$. Therefore, $G_0$ is set by curve-fitting,

$$G_0(s) = \frac{b}{s+a} = \frac{0.74}{s+0.74}. \quad (20)$$

![Figure 7. The Bode plot of the linear dynamics of the SMA-wire.](image)

Moreover, the rise time, settling time, and overshoot with respect to different adaptation gains $\gamma$ are evaluated in simulation. It can be seen from Figure 8 that when the adaptation gain $\gamma$ equals one, the system has the shortest settling time and a relatively small rise time and overshoot. Therefore, $\gamma$ is set to one in our experiment (the result is shown in Figure 9).

![Figure 8. Simulation results of three performance measures with respect to adaptation gains $\gamma$.](image)
In this article a control scheme for integrating system-nonlinearity-reduction and MRAS is proposed and demonstrated via a positioning example with an SMA-wire. The experimental results demonstrated the MRAS robustness to external disturbance and improving the positioning performance. In addition, using the proposed control scheme, the simulation results will closely match the experimental results, which is useful to predict the system performance at the controller-design stage. In the future, a model to capture the difference of dynamics when heating and cooling a SMA will be studied. The proposed scheme may further improve the positioning performance using this new model, and be extended to additional SMA-actuated applications such as micro-manipulator and mimetic hands.

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References


